

専門科目 (TG010001)

# 流体力学II

## Fluid Mechanics II

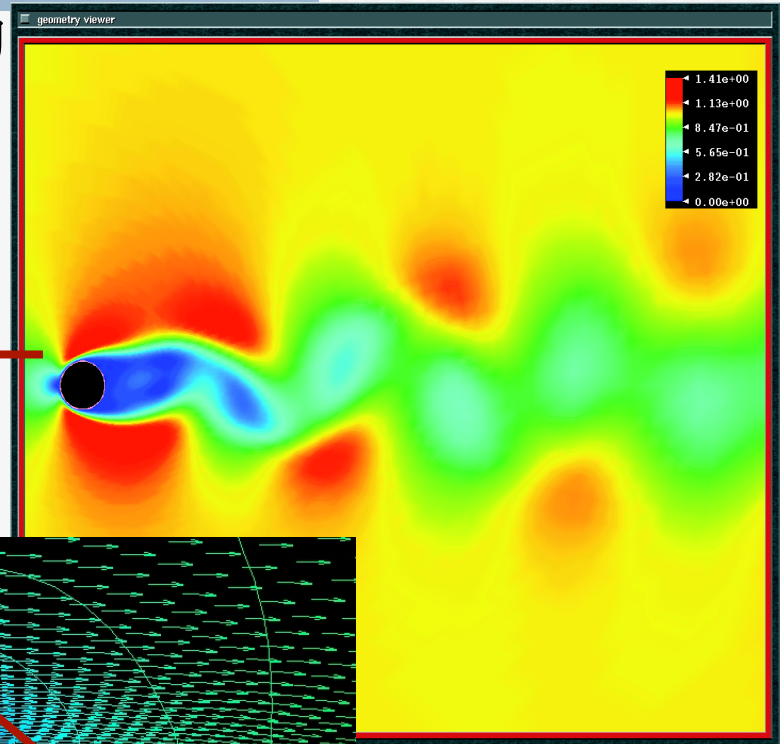
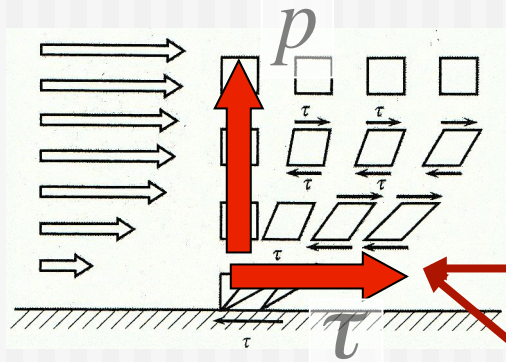
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劉 浩  
太田匡則

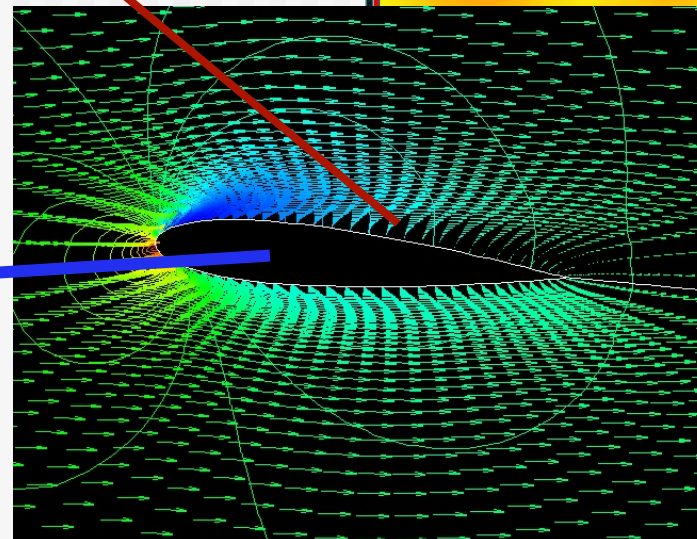
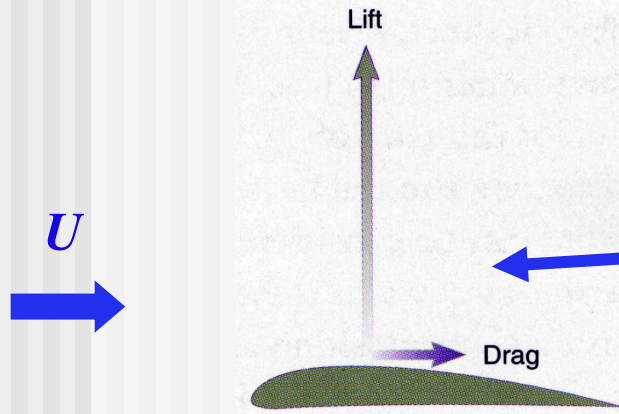
# 物体に働く流体力

## Fluid dynamic forces acting upon a body

- 圧力とせん断応力（摩擦）：微視的



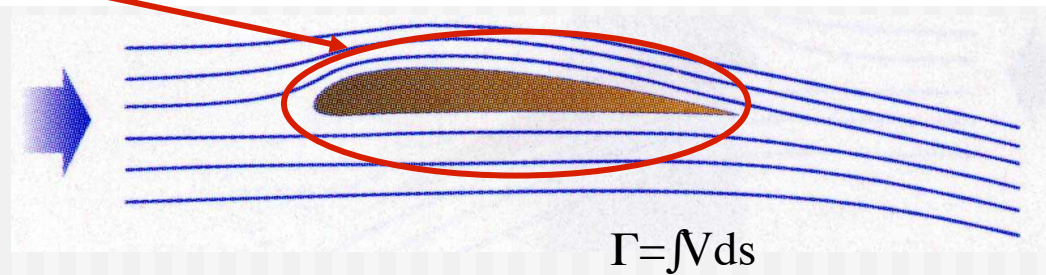
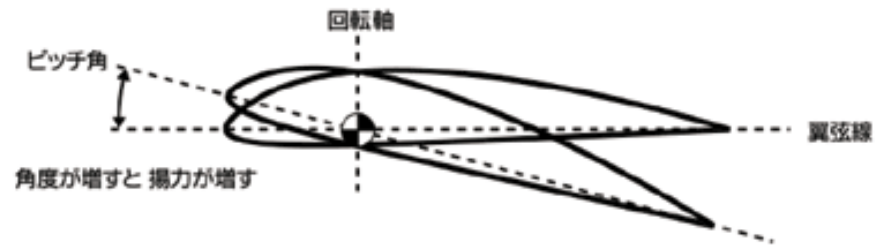
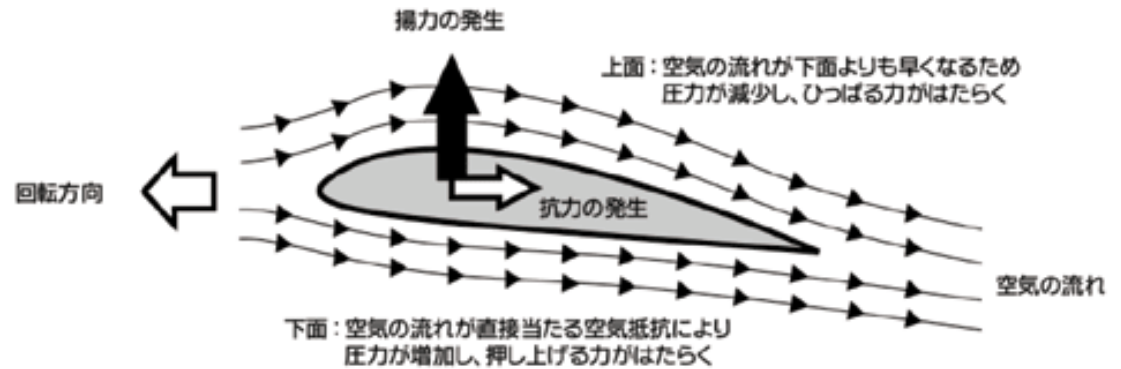
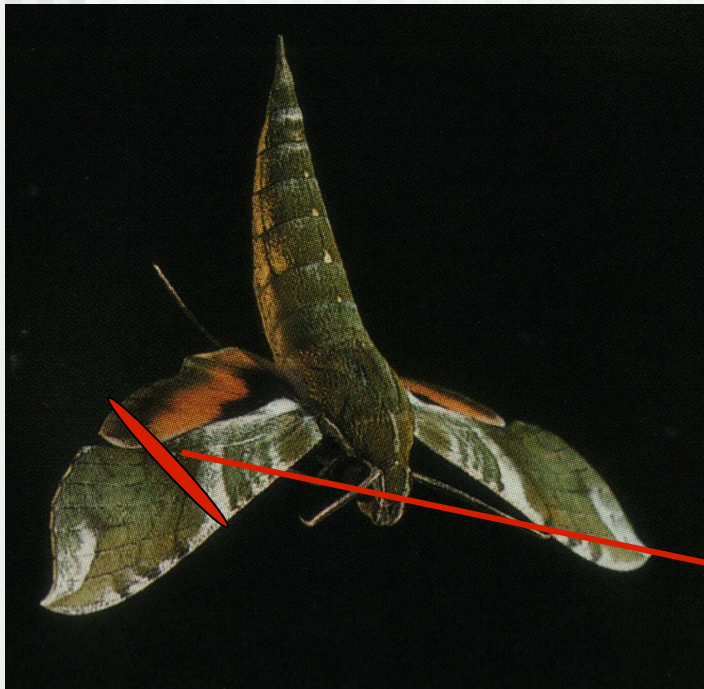
- 揚力と抗力：巨視的



# How can an airfoil generate lift force?

## 揚力の発生メカニズム

Circulation (bound vortex) create a lift to keep a flyer aloft!



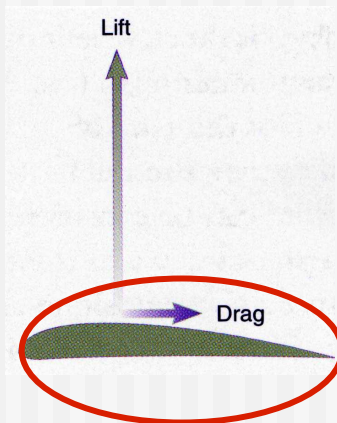
# Bernoulli's Equation: pressure difference over the airfoil

$$\frac{\partial V}{\partial t} + \frac{p}{\rho} + \frac{1}{2} V^2 + gz = C(t)$$

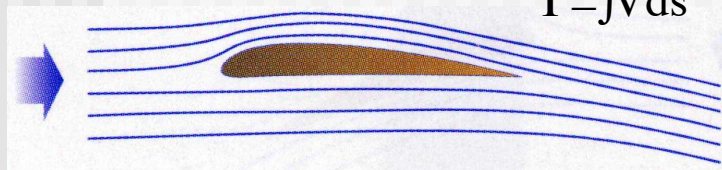
Dynamic pressure:  $\rho V^2/2$

Magnus Effect: inflow+circulation

$$F(\text{lift \& drag}) = \int_S (-p + \tau) dS$$



$$\Gamma = \int V ds$$



# 境界層の解析例 1

## Boundary layer problems

### ■ 平板に沿う境界層 1 :

@境界層内部速度分布の相似

$$\frac{u}{U} \propto f\left(\frac{y}{\delta}\right)$$

$$\delta(x) \propto x / \sqrt{\text{Re}_x}, \text{Re}_x = Ux / \nu$$

@座標変換 (写像) :

$$x' = x, \eta = y \sqrt{\text{Re}_x} / x$$

$$f(\eta) = \Psi \sqrt{\text{Re}_x} / Ux \rightarrow \Psi : \text{StreamFunction}$$

@ブラジウスの微分方程式 :

$$u = \frac{\partial \Psi}{\partial y} = \frac{Udf}{d\eta}, v = -\frac{\partial \Psi}{\partial x} = -\frac{\partial \Psi}{\partial x'} - \frac{\partial \eta}{\partial x} \frac{df}{d\eta} = \frac{1}{2} \frac{U}{\sqrt{\text{Re}_x}} \left( \eta \frac{df}{d\eta} - f \right)$$

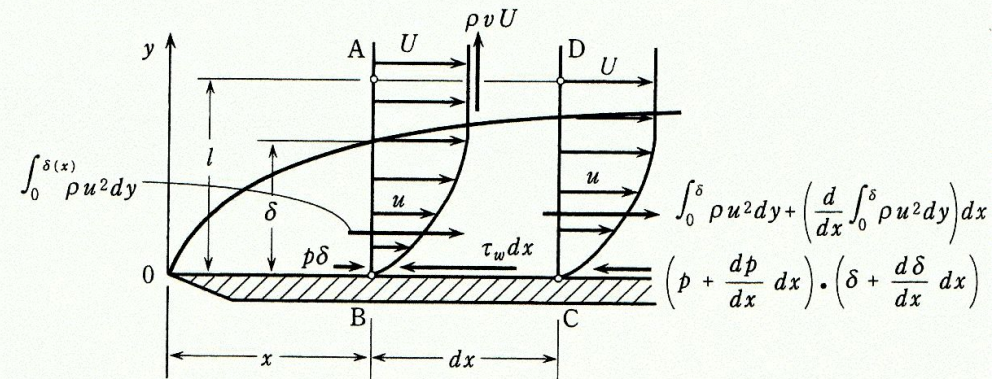
$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} v = U \frac{dU}{dx} + \nu \frac{\partial^2 u}{\partial y^2} \Rightarrow \frac{d^3 f}{d\eta^3} + \frac{f}{2} \frac{d^2 f}{d\eta^2} = 0$$

@境界条件 :

流力2

$$\text{Non-slip} : \eta = 0 \rightarrow f = 0, f_\eta = 0$$

$$\text{Outside} : \eta \gg 1 \rightarrow f_\eta = 1$$



# 平板の抵抗

## Drag on a plate

- 平板に沿う境界層 2 :

@ブラジウスの微分方程式の解

数値積分解は実験値とよく合致

展開近似 :

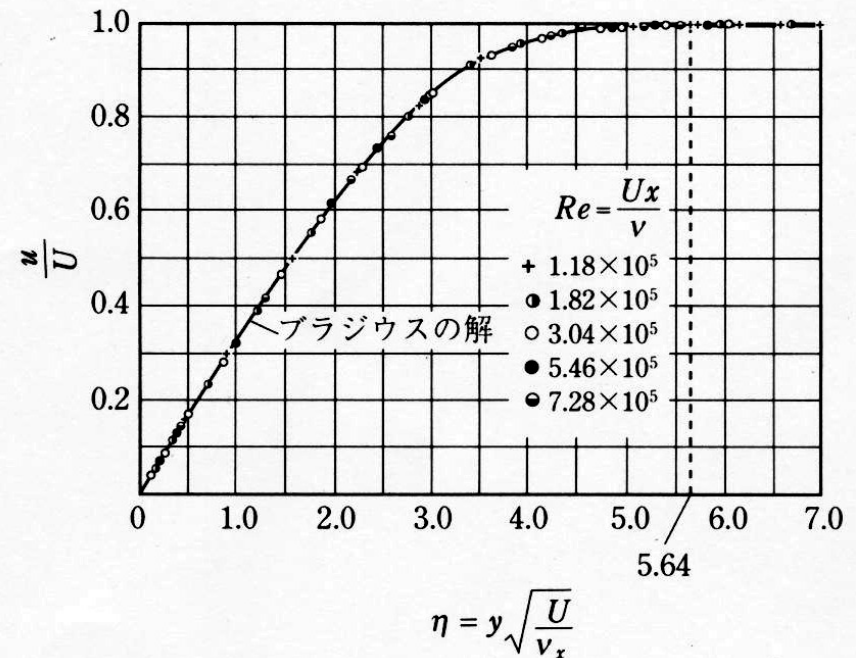
$$u/U = C_0 + C_1\eta + C_2\eta^2 \Rightarrow \delta = 5.48x / \sqrt{Re_x}$$

壁面摩擦抵抗係数 :

$$f_{\eta\eta} = 0.332 \rightarrow \tau_w = \mu \partial u / \partial y$$

$$\tau_w = \mu \sqrt{Re_x} U f_{\eta\eta} / x = 0.332 \rho U^2 / \sqrt{Re_x}$$

$$C_f = \tau_w / (0.5 \rho U^2) = 0.664 / \sqrt{Re_x}$$



@例題6.3:  $U=20\text{m/s}$ ,  $x=30\text{cm}$  のとき、境界層と壁面せん断応力を求めなさい。 $Re_x = Ux/\nu = 3.9 \times 10^5$

$$\delta = 5.48x / \sqrt{Re_x} = 2.63\text{mm}, \tau_w = 0.332 \rho U^2 / \sqrt{Re_x} = 248\text{g} / \text{ms}^2$$

# 円柱の抵抗

## Drag on a circular cylinder

- @円柱まわりの境界層方程式の解：

外縁でポテンシャル流れ

$$U(x) = U \sin\left(\frac{x}{a}\right) = 2U_0 \left(\frac{x}{a}\right) - 2U_0 \frac{1}{3!} \left(\frac{x}{a}\right)^3 + \dots$$

円柱表面で境界層方程式

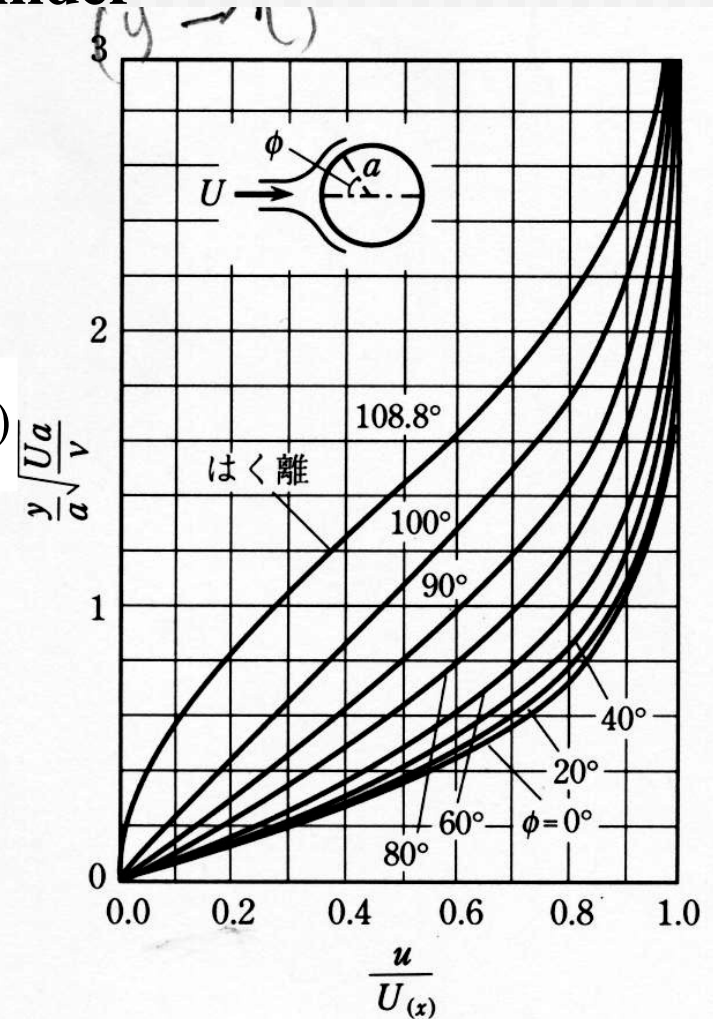
$$f(\eta, \frac{x}{a}) = f_1(\eta, \frac{x}{a}) + f_3(\eta, \frac{x}{a})^3 + \dots \Rightarrow f_i (i = 1, 3, 5, \dots)$$

$$\frac{u}{U(x)} \Leftrightarrow \frac{y}{a} \sqrt{\frac{Ua}{\nu}}, \phi(0 \rightarrow \pi)$$

@流れの剥離：

境界層は上流側で薄く、下流側で厚く  
 角度 = 108.8 (実験では80) になると、  
 剥離が発生

$$u = 0, \frac{\partial u}{\partial y} < 0$$



# 球の抵抗

## Drag on a sphere

- 円柱と球のまわりの境界層：  
@球まわりの境界層方程式の解：

外縁でポテンシャル流れ

$$U(\phi) = 3U \sin\left(\frac{\phi}{2}\right) = 6U_0 \left(\frac{\phi}{2}\right) - 6U_0 \frac{1}{3!} \left(\frac{\phi}{2}\right)^3 + \dots$$

円柱表面で境界層方程式

$$f(\eta, \frac{\phi}{2}) = f_1(\eta, \frac{\phi}{2}) + f_3(\eta, \frac{\phi}{2})^3 + \dots \Rightarrow f_i (i = 1, 3, 5, \dots)$$

$$\frac{u}{U(x)} \Leftrightarrow \frac{y}{a} \sqrt{\frac{Ua}{\nu}}, \phi(0 \rightarrow \pi)$$

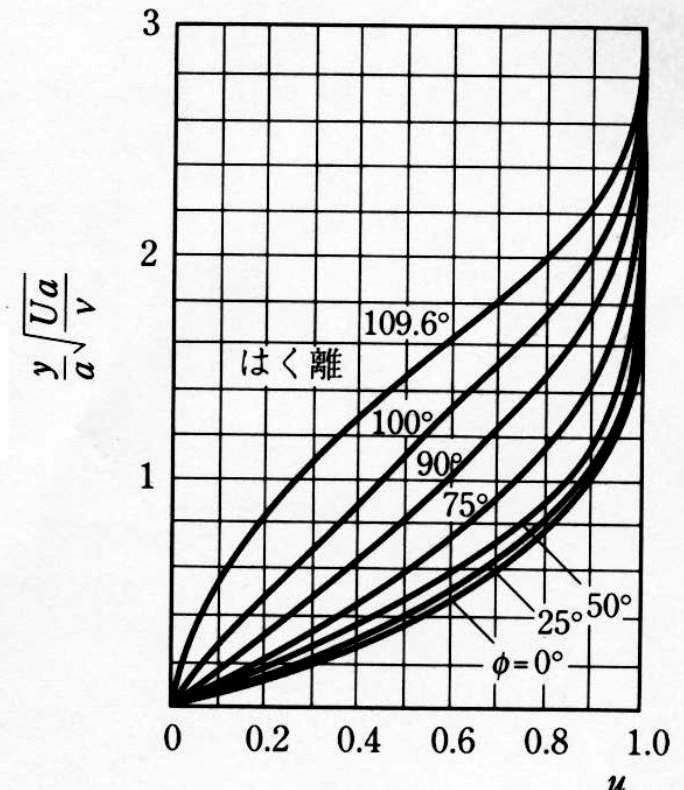
- @流れの剥離と伴流：

境界層は上流側で薄く、下流側で厚く

角度 = 109.6 (実験では84) になると、  
剥離が発生

流力2

$$u = 0, \frac{\partial u}{\partial y} < 0$$



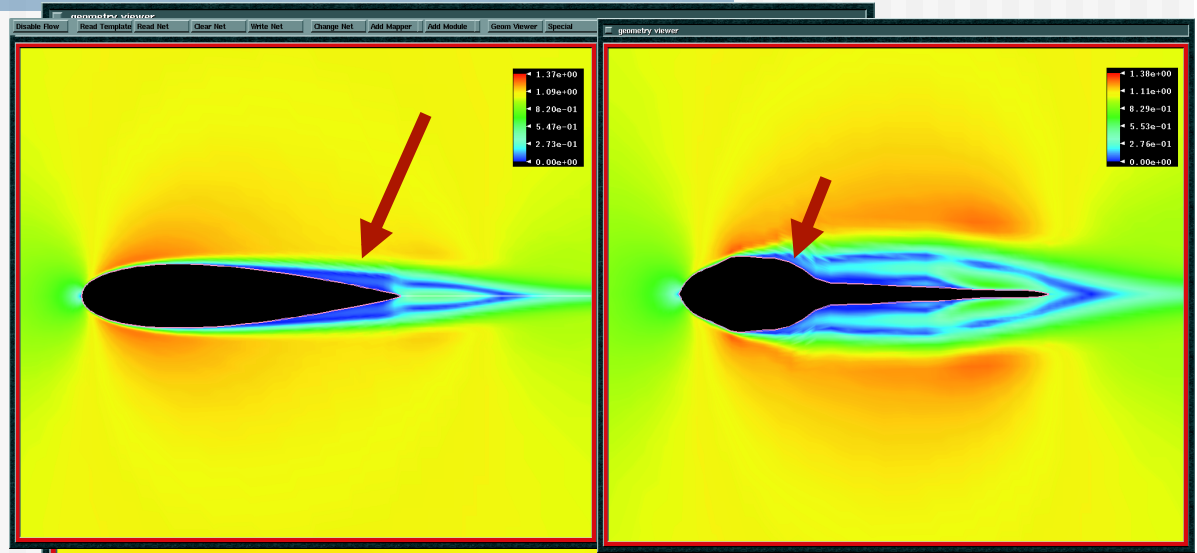


# 境界層に伴う剥離

## Boundary layer problems

- 境界層、剥離、後流：  
@ポテンシャル流れでは  
エネルギー保存により  
前後縁では圧力対称、  
剥離無し

@粘性があると、摩擦による  
エネルギー損失が生じ  
後縁付近では逆流が発生

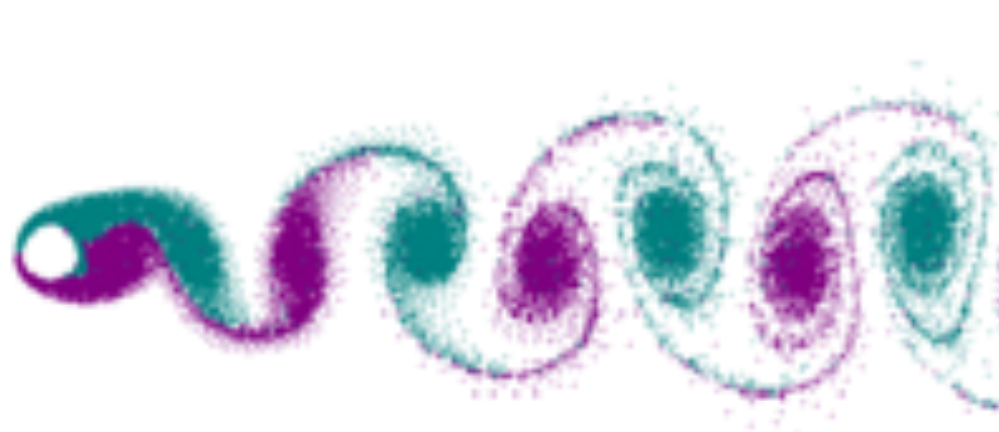


$$p + \frac{\rho}{2} U^2 = C \rightarrow \mu \frac{d^2 u}{dy^2} = \frac{dp}{dx}$$

@剥離点：

$$u = 0, \frac{\partial u}{\partial y} < 0$$

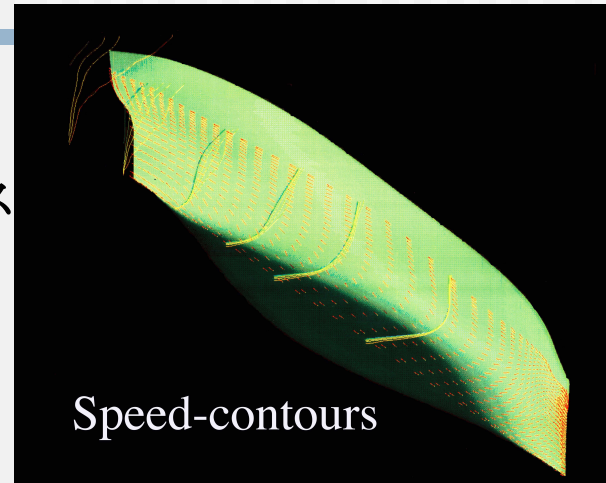
流力2



# 境界層に伴う剥離

## Boundary layer problems

- 境界層、剥離、後流：  
@理想流体では抗力はゼロ：  
ダランベールのパラドックス



- @解決方法：粘性を考慮する
  - 1) 境界層近似解
  - 2) ナビエーストークス方程式の数値解

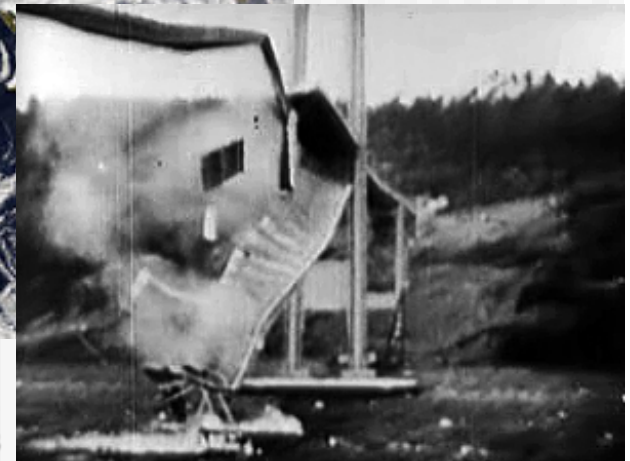
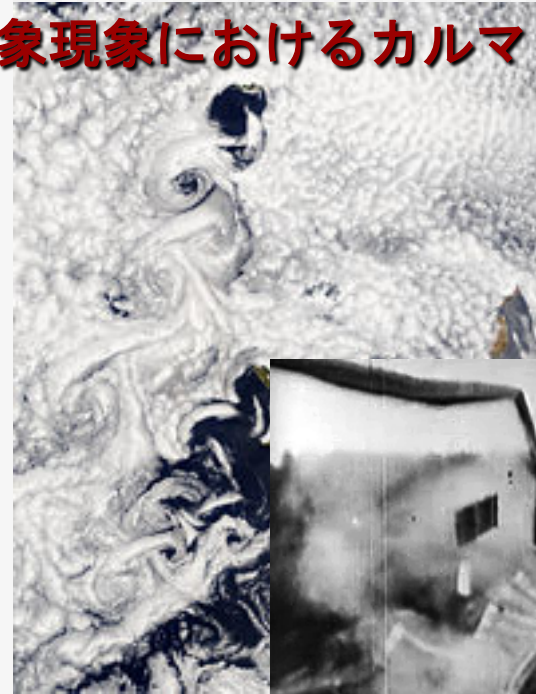
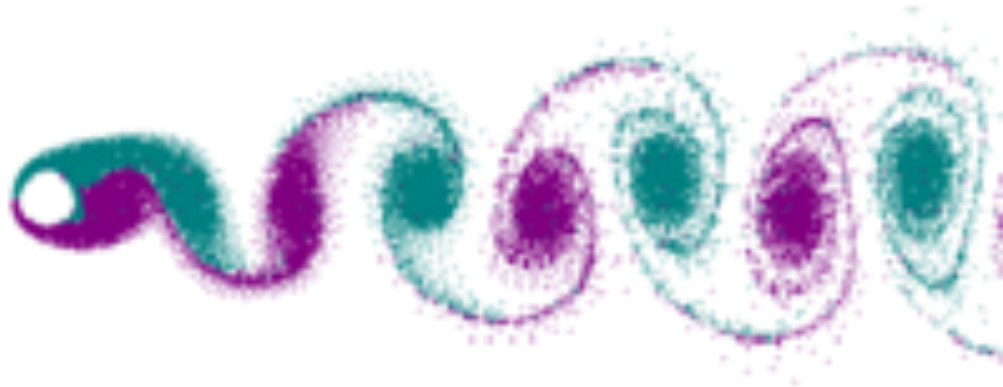
# 円柱の抵抗2

## Drag on a circular cylinder

### ■ 2次元円柱まわりの流れ：カルマン渦

カルマン渦、またはカルマン渦列は、流れのなかに障害物を置いたとき、または流体中で固体を動かしたときにその後方に交互にできる渦の列のことをいう。ハンガリー人の流体力学者セオドア・フォン・カルマンに因んでいる。

### 気象現象におけるカルマン渦

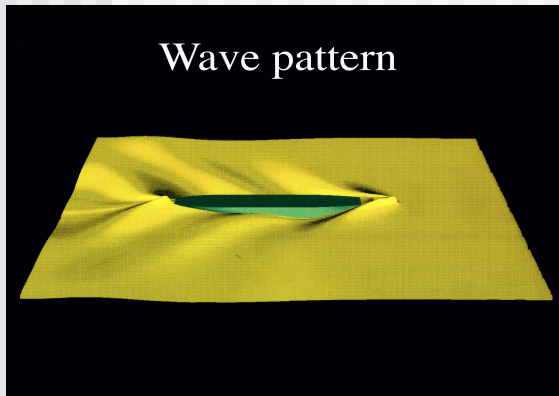


# 円柱の抵抗2

## Drag on a circular cylinder

### ■ 抵抗とは：前進方向に反する力

@全抵抗=圧力（形状）抵抗+摩擦抵抗+波動（非定常）抵抗



@表面の圧力分布：

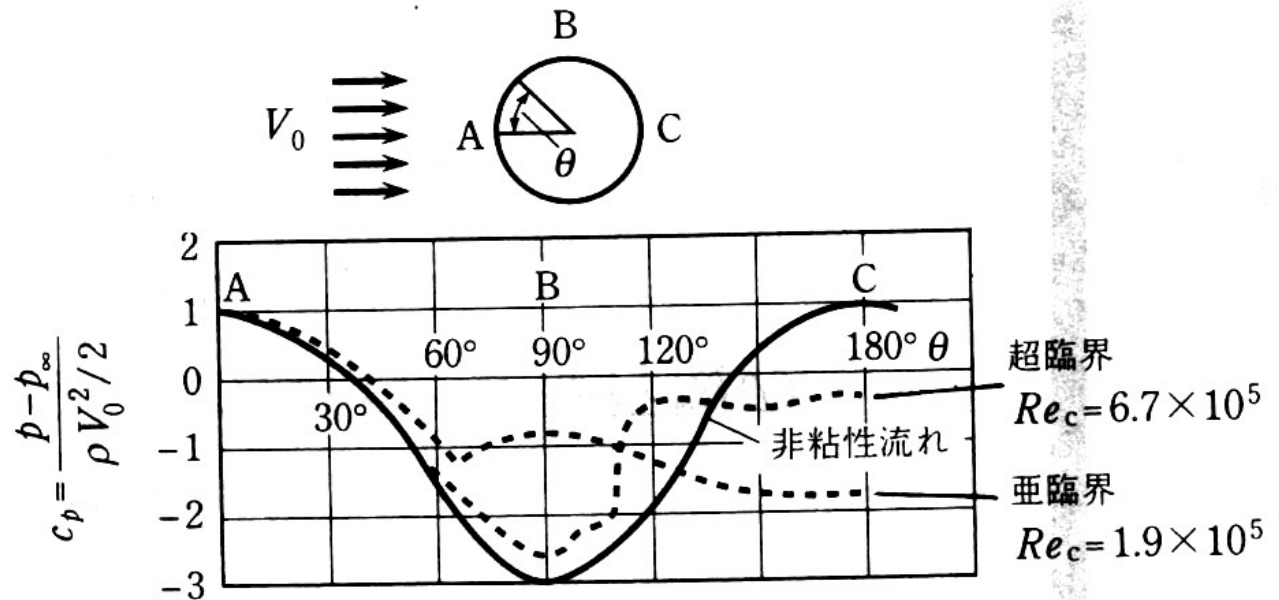
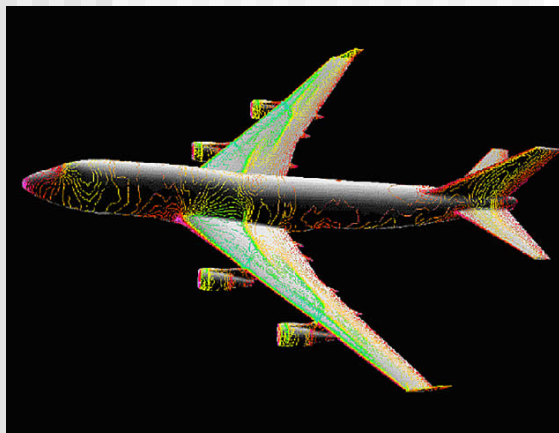
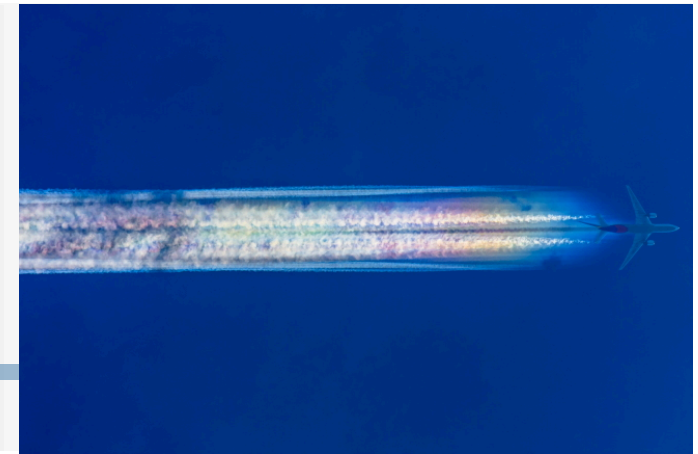


図 6・18 円柱表面の静圧分布

# 円柱の抵抗 3

## Drag on a circular cylinder



### ■ 抵抗とは：前進方向に反する力

@レイノルズ数依存性：

@層流と乱流：

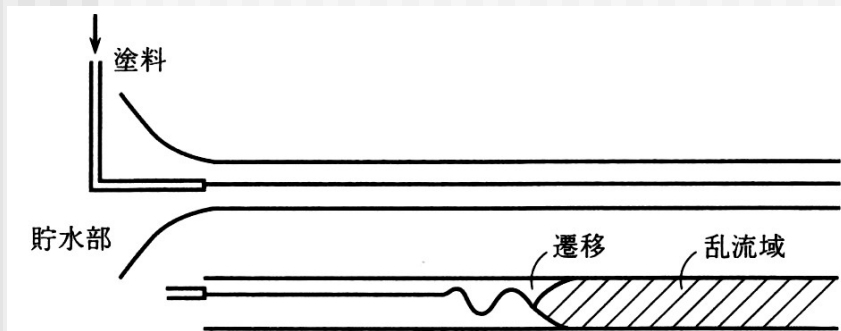
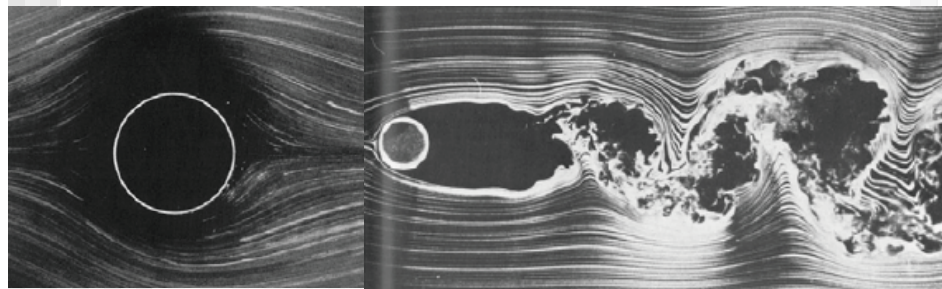
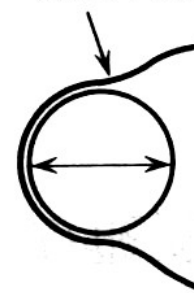


図 6・31 レイノルズの実験

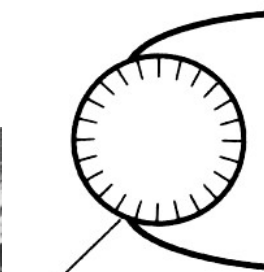


流力2

粘性域は境界層と後流の渦内部のみ

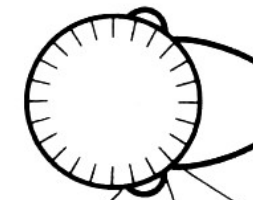


(b)



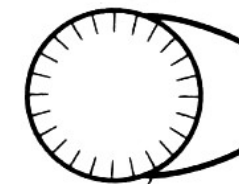
層流はく離

(c)  $Re < 3 \times 10^5$



層流はく離  
乱流はく離  
乱流再付着

(d)  $3 \times 10^5 < Re < 3 \times 10^6$



乱流はく離

(e)  $Re > 3 \times 10^6$

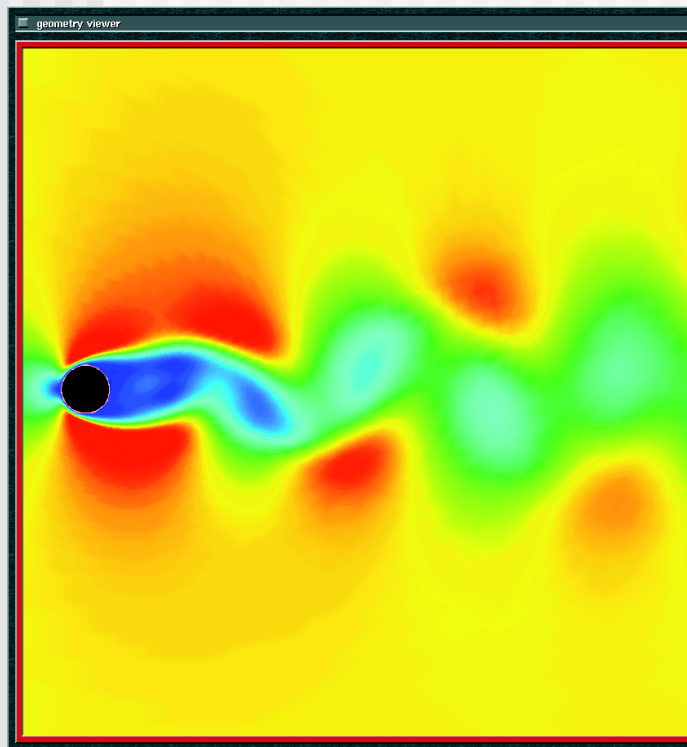
# 円柱の抵抗 4

## Drag on a circular cylinder

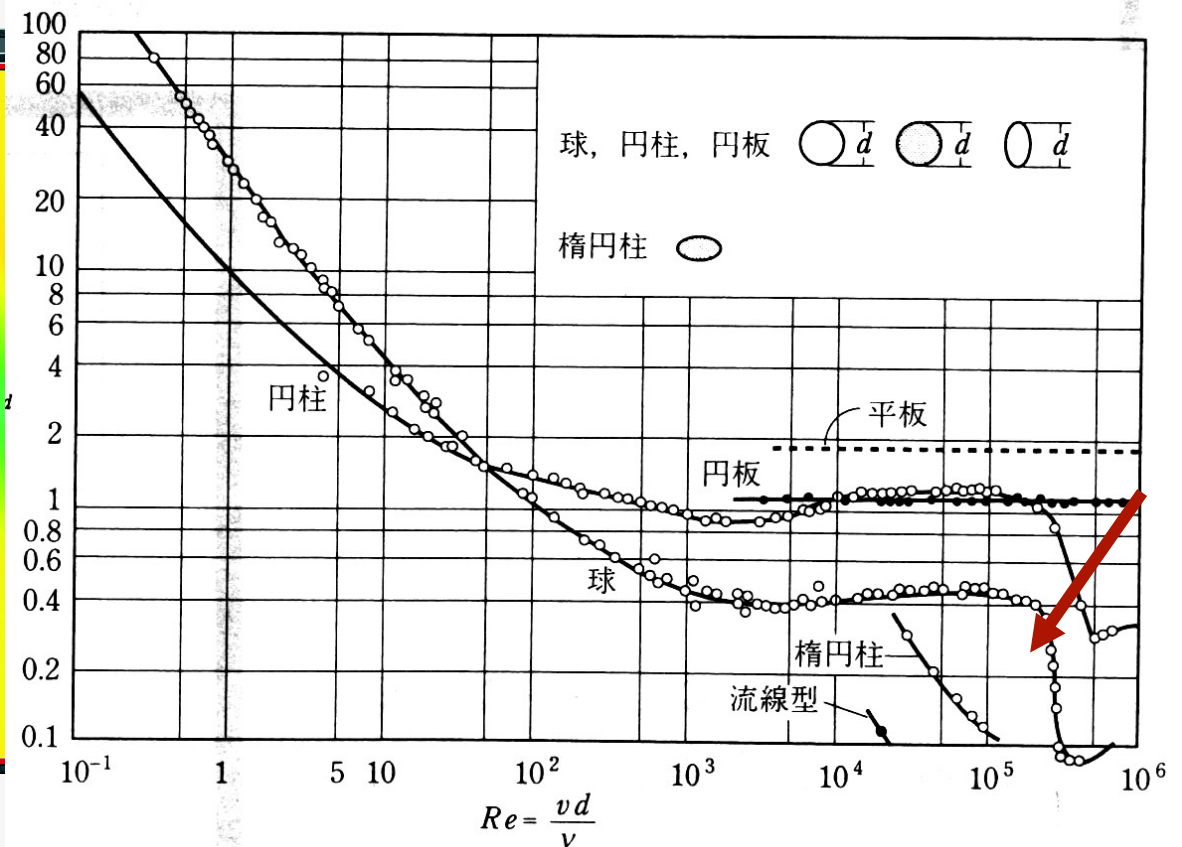
### ■ 抵抗とは：前進方向に反する力

@臨界レイノルズ数： $Re_c$ (Critical Reynolds number)

層流が乱流に遷移するときの臨界を表す



流力2



# 円柱の抵抗4

## Drag on a circular cylinder

### ■ カルマン渦列： von Karman Vortex Street(Sheet)

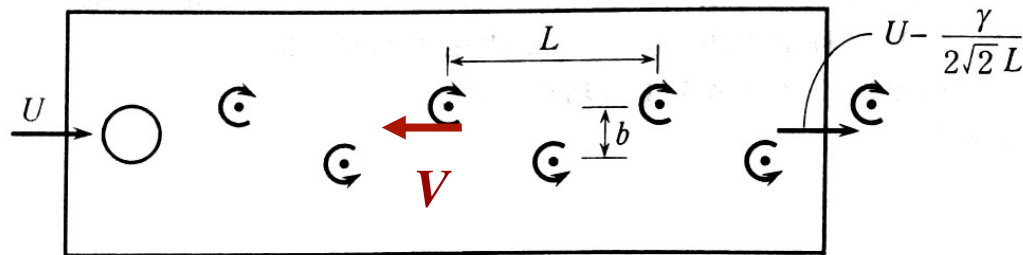
@対称と非対称の渦列

Reの増加とともに、渦放出の周波数n、強さ及び後流の幅が増加

@カルマンの渦列速度： 電線の騒音など安定(Re=65)：

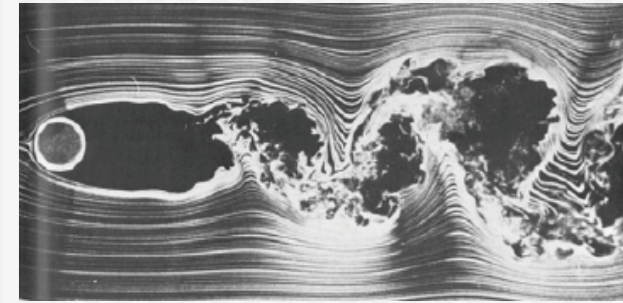
$$\cosh(\pi b / L) = \sqrt{2} \rightarrow b / L = 0.2806$$

非安定：  $V = \frac{\gamma}{2\sqrt{2}L} \rightarrow U - V = U - \frac{\gamma}{2\sqrt{2}L}$



ストローハル数：  $St = \frac{nD}{U}$

流力2



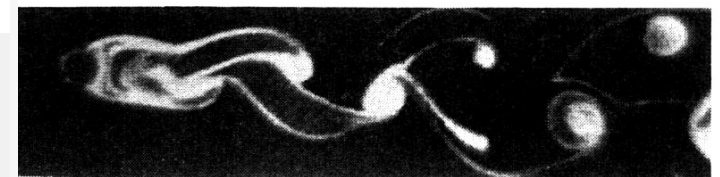
Re = 102



Re = 161



Re = 225



Re = 281

# 平板の抗力

## Drag on a flat plate

- 平板に沿う境界層：  
@ブラジウスの微分方程式の解

壁面摩擦抵抗係数：

$$\tau_w = \mu \sqrt{Re_x} U f_{\eta} / L = 0.332 \rho U^2 / \sqrt{Re_x}$$

$$C_f = \tau_w / (0.5 \rho U^2) = 0.664 / \sqrt{Re_x}$$

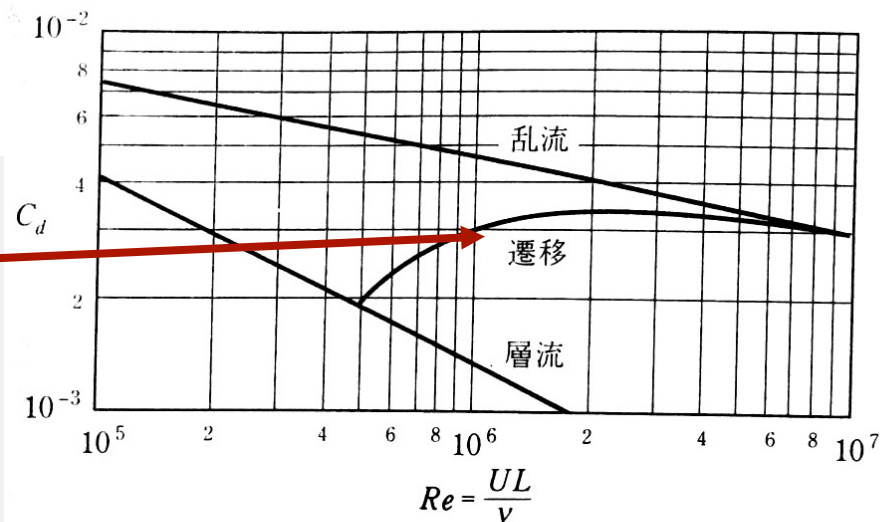
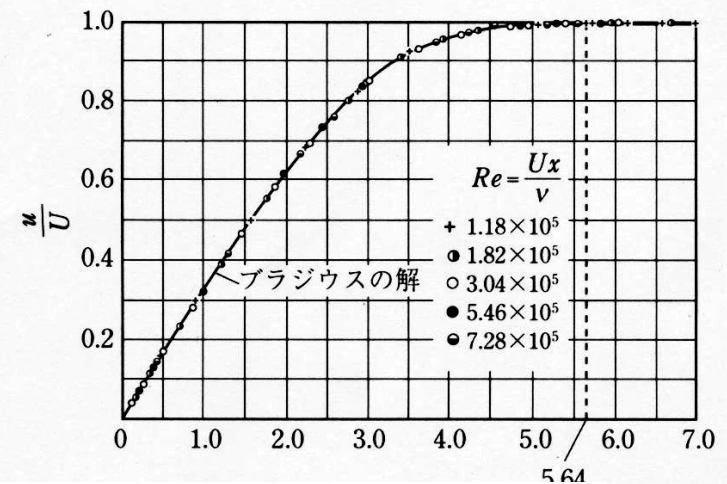
$$\Rightarrow C_d = \int_0^L \frac{C_f}{L} dx = 1.328 / \sqrt{Re}$$

- @レイノルズ数依存性：

$$C_d = 0.074 / Re^{0.2}$$

- @一様流に垂直な平板  
～一般形状の物体

流力2



層流  $C_d = \frac{1.328}{\sqrt{Re}}$ , 遷移  $C_d = \frac{0.074}{Re^{1/5}} - \frac{1700}{Re}$ , 乱流  $C_d = \frac{0.074}{Re^{1/5}}$



# 円管内の抵抗1

## Drag inside a tube: 3D Hagen-Poiseuille Flow

- 流量、断面平均流速、摩擦応力と係数：

$$Q = \int_0^R w \cdot 2\pi r dr = \int_0^R \frac{2\pi r}{4\mu} \frac{dp}{dz} (r^2 - R^2) dr = -\frac{\pi}{8\mu} \frac{dp}{dz} R^4$$

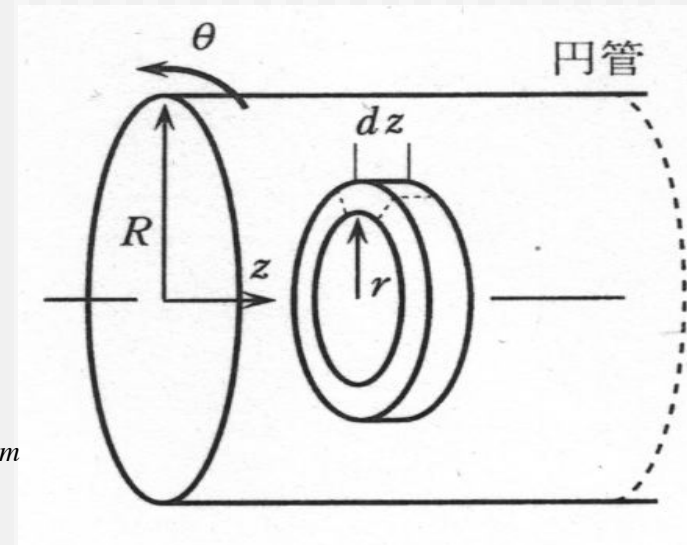
$$U_m = \frac{Q}{\pi R^2} = -\frac{1}{8\mu} \frac{dp}{dz} R^2$$

$$Re = \frac{\rho U_m (2R)}{\mu}$$

$$\tau_0 = \mu \frac{dw}{dy} = -\mu \left. \frac{dw}{dr} \right|_{r=R} = -\frac{1}{2} \frac{dp}{dz} R = \frac{4\mu}{R} U_m$$

$$C_f = \frac{\tau_0}{0.5 \rho U_m^2} = \frac{8\mu}{\rho R U_m} = \frac{16}{Re}$$

流力2



# 円管内の抵抗1

## Drag inside a tube

### ■ 円管内の抵抗：

@低レイノルズ数流れ：

$$\tau_0 = \mu \frac{dw}{dr}_{r=R} = -\frac{1}{2} \frac{dp}{dz} R = \frac{4\mu}{R} U_m = \frac{8}{Re}$$

$$f = \frac{\tau_0}{(\rho/2)U_m^2} = \frac{8\mu}{\rho R U_m} = \frac{16}{Re}, Re = \frac{U_m(2R)}{\mu/\rho}$$

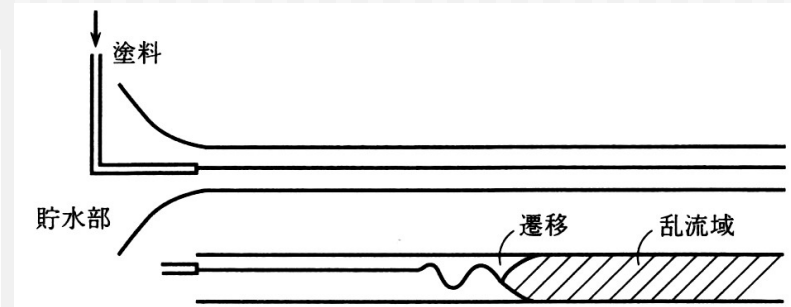


図 6・31 レイノルズの実験

@高レイノルズ数流れ：

助走空間：

$$L/D = 0.065 Re$$

$$C_d = 0.332 / Re^{1/2}$$

完全発達流：

乱流：Re>2000

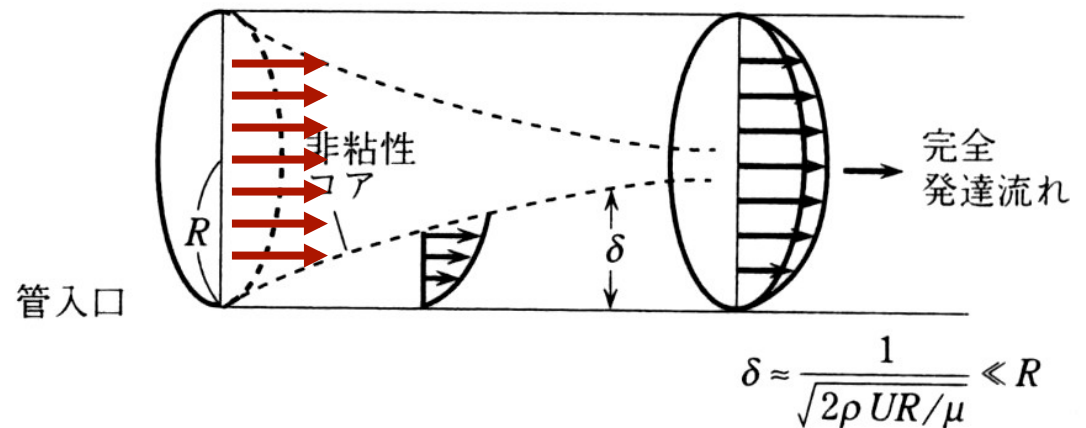


図 6・30 管内の境界層の発達

# 円管内の抵抗2

## Drag inside a tube

### ■ 円管内の抵抗：

@乱流境界層と壁面抵抗：

1/7乗則（経験式）： $Re < 1 \times 10^7$

対数速度分布（経験式）： $Re > 1 \times 10^7$

$$\frac{v}{v_{ret}} = 8.56 \left\{ \frac{\rho(R-r)v_{ret}}{\mu} \right\}^{1/7}$$

$$v_{ret} = (\Delta p R / 2 \rho L)^{1/2}$$



$$C_f = \frac{D\Delta p}{4L\rho v^2 / 2} = \frac{0.0791}{Re^{1/4}}$$

ブラジウスの抵抗の式：

流力2

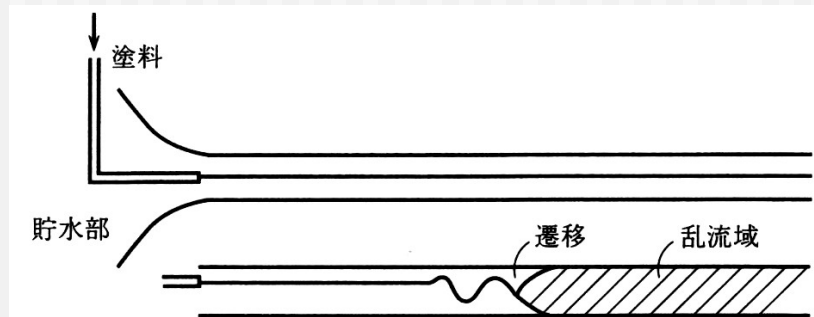
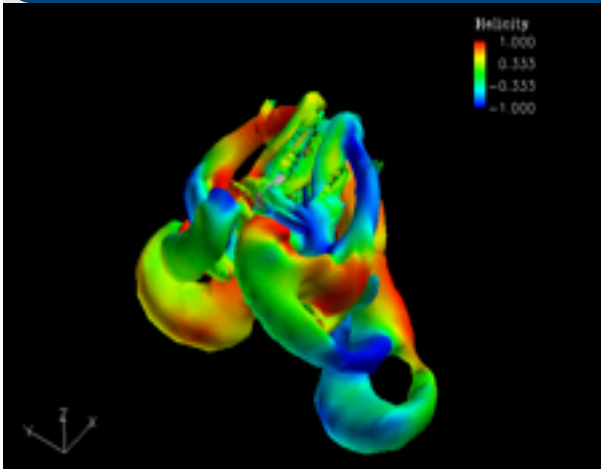


図 6・31 レイノルズの実験

## Flapping-wing Aerodynamics



# Computational Biomechanics for Bio-inspired Flight System

Fluid dynamics

Dynamics & Control

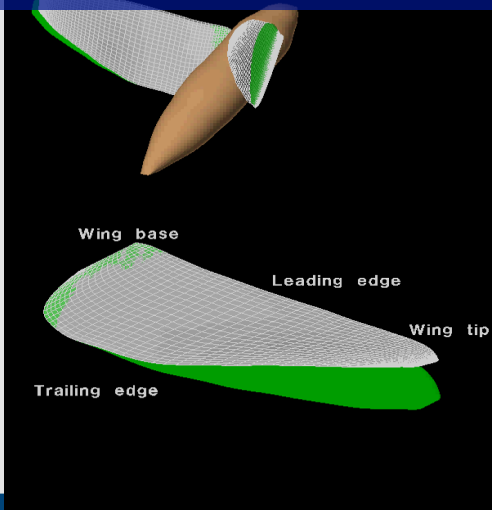
## Flight Dynamics & Flight Control



1<sup>st</sup> versatile simulator for biology-inspired flight system

Structural mechanics

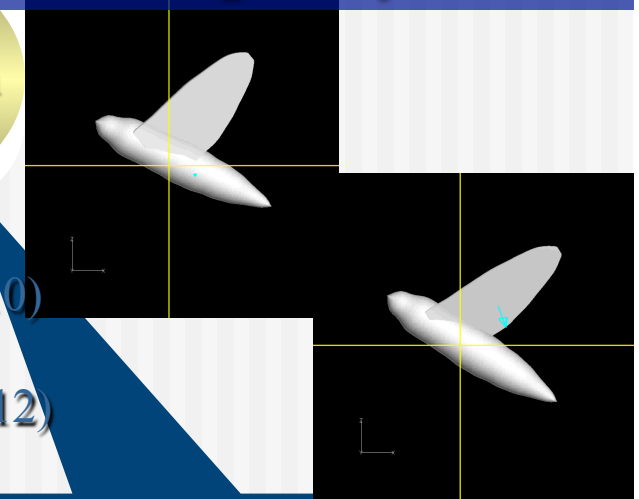
Optimization



Fluid-Structure Interactions (FSI)

M. Maeda & H. Liu (*JBMECH* 2010)  
Y. Nakata & H. Liu (*JCP*, 2011;  
*Proc R. S. B*, 2011; *B&B* 2012)  
N. Gao & H. Liu (*JTB* 2010)  
H. Liu, *JCP* (2009)  
H. Liu & H. Aono, *B&B* (2009)  
H. Aono & H. Liu, *JEB* (2008)

Wing shape & Kinematics Optimization

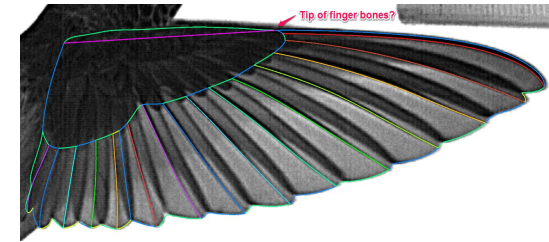
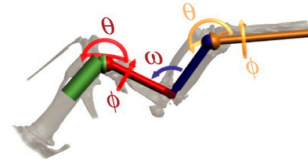


1<sup>st</sup> realistic CFD model with flexible wings revealed importance of wing deformation in hummingbird hovering.

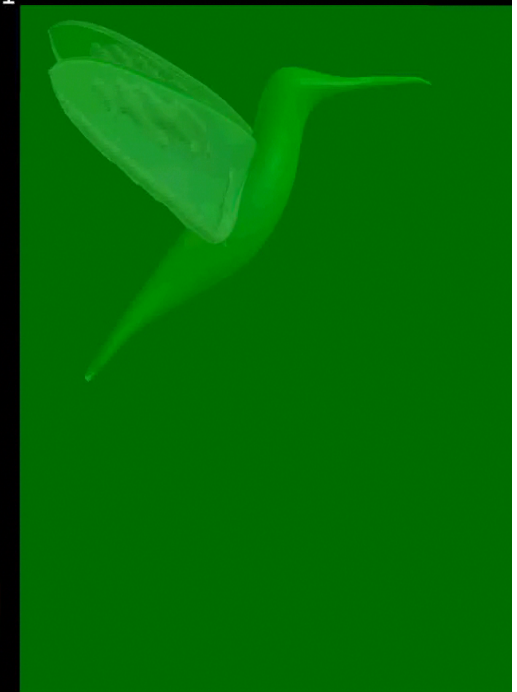
(Maeda&Liu, submitted 2015)



x 0.015 speed



$t/T = 0.01$



Wing morphology and deformation in hummingbird can augment aerodynamic forces and achieve better aerodynamic efficiency.